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**APPLICATION OF THE PRINCIPLE OF CORRESPONDING
STATES TO TWO-PHASE CHOKED FLOW**

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APPLICATION OF THE PRINCIPLE OF CORRESPONDING STATES TO TWO-PHASE CHOKED FLOW

Robert C. Hendricks and Robert J. Simoneau

Introduction

The state of research in two-phase choked flow, while substantially developed in recent years, still has significant unanswered questions. Hsu (ref. 1) presents a good review on this. The proper choice of fluid mechanic and thermodynamic models is not totally resolved (refs. 2 and 3). Henry and Fauske (ref. 3) make a strong argument that nonequilibrium effects must be considered; however, in doing so they are forced into empiricisms. As a result, designers are reluctant to work with the theories beyond the range of known data, and frequently demand data for the fluid they are using and in the range they are working. In the case of oxygen this is particularly distressing since testing is expensive and hazardous.

With this in mind the present paper examines the possibility of applying the principle of corresponding states to two-phase choked flow. If this is valid, designers, using data from other fluids to back them up, could apply the theory of their choice to fluids where there is little or no data, such as oxygen.

In ref. 4 it was noted that, while the isenthalpic Joule-Thomson coefficient (μ) is not a corresponding states parameter, a reducing parameter ($\mu_c \approx T_c/6P_c$) could be defined and several figures of $\mu_R = \mu/\mu_c$ as a function of $T_R = T/T_c$ with $P_R = P/P_c$ as a parameter were presented. Several fluids including methane, oxygen, and nitrogen appear to form an average parametric plot which indicates that μ must nearly obey the principle of corresponding states as discussed in ref. 4. With this in mind, it was assumed that there could be several thermodynamic flow processes which nearly obey the principle and consequently could be placed in some reduced form. It was decided to examine whether two-phase choked flow could be one of them.

2.

As a cautionary note, the authors wish to emphasize that in this paper they are not advocating any particular theoretical analysis for two-phase choked flow. In fact, the authors believe that when the final verdict is in, it will involve a combination of theories applicable in various regions. The purpose here is simply to evaluate corresponding states normalization.

ANALYSIS

While the arguments leading to this conclusion are beyond the scope of this paper, it is well known that one-dimensional isentropic flow through a nozzle is related to the enthalpy difference. Since the enthalpy difference nearly obeys the corresponding states principle, it follows that nozzle flow should also nearly obey the principle. Such is the case even when fuel rich combustion is taking place as in a rocket engine, a well known result in rocketry.

Critical Flow Rate

The maximum flow rate (choked flow) of a gas through a nozzle has been shown by several authors (eg. ref. 1, 2, 5) to be a thermodynamic parameter dependent on the state points (P_0, T_0) and (P_t, T_t) which satisfy equation 1

$$\begin{aligned} H_0 - H_t &= 10^{-7} \frac{C_t^2}{2} \\ G_{max} &= P_t C_t \end{aligned} \quad (1)$$

In reference 6, the basic flow equation for both nonequilibrium and equilibrium models is given as

$$G^2 = -\frac{2}{V^2} \int_{P_0}^P V dP \quad (2)$$

and the maximum flow rate was found by maximizing equation 2

$$G_{max}^2 = - \left[x \frac{dV_g}{dP} + (1-x) \frac{dV_l}{dP} + (V_g - V_l) \frac{dx}{dP} \right]_t^{-1} \quad (3)$$

3.

By simply normalizing equation 2, the reduced maximum flow rate becomes:

$$G_R = \frac{G_{max}}{G^*} = \frac{G_{max}}{\sqrt{\rho_c P_c}} \quad (4)$$

If, however, the enthalpy form of the flow equation, equation (1), is used, the normalizing constants are somewhat different. In arguing that the enthalpy nearly obeys the principle, the reduced enthalpy was found to be

$$\frac{H - H^0}{RT_c} = f(T_R, P_R) + g(T, P) \quad (5)$$

where f and g are rather complex functions and f obeys the principle while g does not. Examination of equation (1) in light of the reduced enthalpy of equation (5) suggests the proper normalizing parameter to be

$$G^* = \sqrt{\frac{\rho_c P_c}{Z_c}} \quad (6)$$

Clearly for the near spherical molecules, methane, nitrogen, argon and oxygen, it makes little difference which form of G^* is used; however equation 6 should be the preferred form for G^* . The critical constants used herein are from reference 8 and given as Table I.

Critical Pressure Ratio

The isentropic expansion of an ideal gas through a nozzle leads to the critical pressure ratio being

$$\frac{P_c}{P_0} = \left[\frac{2}{\gamma + 1} \right]^{\frac{\gamma}{\gamma - 1}} \quad (7)$$

Equation (7) is not reduceable by corresponding states since C_p , C_v and γ are not corresponding states variables. Consequently this would lead one to believe that the critical pressure ratio cannot be reduced, since the asymptotic value cannot be reduced. On the other hand, if the low density ($\rho \rightarrow 0$) background values, C_{p_0} and C_{v_0} are subtracted out then $C_p - C_{p_0}$ and $C_v - C_{v_0}$ can be nearly

4.

reduced by corresponding states. This is discussed by Francis and Luckhurst (ref. 7). Such results suggest that γ in equation (7) could be reduced in similar manner. The authors' efforts to subtract out this background and normalize γ were not successful. However, it was noted that in the near critical region there existed a correspondence between the critical pressure ratios for nitrogen and methane for the same reduced pressure and temperature. If the principle is to work, it should work in the near critical where background effects become small.

It was decided then to examine the reduced critical pressure ratio for both cases: i) the dense fluid region where the normalizing parameter is unity

$$\left. \frac{P_t}{P_o} \right)_R = \frac{P_t}{P_o} \quad (\text{dense fluid}) \quad (8)$$

and ii) the gas region where the normalizing parameter is the perfect gas asymptote, eq. 7,

$$\left. \frac{P_t}{P_o} \right)_R = \frac{P_t/P_o}{(P_t/P_o)_{PG}} = \frac{P_t/P_o}{\left[\frac{2}{\gamma+1} \right]^{\gamma/(\gamma-1)}} \quad (\text{gas}) \quad (9)$$

The asymptotic values $(P_t/P_o)_{PG}$ used herein are given as Table II. This is an ad hoc action which will be discussed further in the results.

RESULTS

Comparison with Isentropic Equilibrium Theory

Equation 2 was solved assuming an isentropic equilibrium expansion and maximized to give the critical flow rate (G_{\max}) and critical flow pressure ratio (P_t/P_o) for the corresponding states fluids nitrogen, oxygen, argon, and methane. The thermodynamic properties were taken from reference 8. The choice of the equilibrium model was made because the calculation is very straightforward, because no empiricisms are involved and because it seems to work well in the thermodynamic

critical region, where the authors are especially concerned. A better choice of models may be appropriate in some regions, especially when P_0 is near saturated liquid, but the equilibrium model will illustrate the corresponding states question. The normalization follows equations 6 and 8 or 9.

Critical flow. - By systematically varying the stagnation pressure, holding stagnation temperature constant, one can produce a map of reduced critical flow rate as a function of reduced pressure with reduced temperature as a parameter. Such a map of four corresponding states fluids is given as figure 1. A closer examination of fig. 1 indicates the calculated values for nitrogen and oxygen are the same; however the values for argon and methane tend to deviate, higher and lower respectively. Furthermore the departure from a single isotherm increases from 2% for $T_R \leq 1.0$ to 10% at $T_R = 1.1$. The substantial increase between $T_R = 1.0$ and $T_R = 1.1$ is at this point unexplained. The departure for $T_R \geq 1.2$ appears to be about 13%. This degree to which the four fluids collapse to a single isothermal curve both strengthens and weakens the corresponding states assumption. It is strengthened in that it works well for $T_R \leq 1.0$. It is disturbing, however, that there is a shift between $T_R = 1.0$ and $T_R = 1.1$, a region where the background (or noncorresponding states effects) should be small relative to the total and should subtract out.

The reader should be reminded that, even though the flow normalization has not perfectly collapsed all the fluids to a single reduced curve, the normalization by G^* has brought fluids with widely varying flow rates for the same reduced conditions very close together. For example at $P_{R0} = 3.0$ and $T_R = 1.1$ the values of G_R for argon and methane are 1.34 and 1.22 respectively, a 9.3% spread; whereas the values of G are 12,600 and 6,200 gm/cm²sec respectively, a factor of 2 difference.

The abnormal small "bumps" in these curves (such as at $T_R = 1.0$, $1.0 < P_R < 1.2$) represent changes in the flow patterns which are best explained in terms of pressure ratio; therefore, discussion is deferred to the next section, where the "bumps" are more pronounced.

Critical flow pressure ratio. - The critical flow pressure ratio normalized by equation 9 as a function of reduced stagnation pressure with reduced stagnation temperature as a parameter is illustrated for argon, oxygen, nitrogen and methane in figure 2. Only the oxygen curves are plotted for values of $T_R > 1.1$ to avoid confusion, since the curves are getting close together and they are folding over. As with the critical flow rate the general observation can be made that representing critical flow pressure ratio as a function of reduced stagnation pressure and temperature tends to bring the four fluids together. While this is very encouraging, there is still a 10-15% spread in some areas, just as with the flow rate curves. Again the $T_R = 1.1$ isotherm shows substantial deviation. It is interesting to note, however, that if $(P_t/P_o)_R$ is plotted with the dense fluid normalization, equation 8, the deviation picture reverses as shown in figure 3. In the dense fluid region and near the thermodynamic critical region, ($T_R = 1.0$, $1.0 \leq P_{R0} \leq 3.0$) the $(P_t/P_o)_R$ curves for the four fluids collapse very nearly to a single curve as a function of reduced pressure and temperature. On the other hand, in the more gaseous regions ($T_R = 1.4$ or $P_{R0} < 1.0$) the spread appears again. Obviously one could contrive a more empirical normalization which would cover both regions, however, since the whole approach on pressure ratio is somewhat ad hoc the authors prefer not to do this and leave it to the discretion of the reader.

As mentioned before, the "bumps" in these curves represent changes in flow patterns. For example, the peak in the $T_R = 1$ isotherm corresponds to the first appearance of net quality in the nozzle. The inflection point near $P_{R0} = 1.02$ occurs when the stagnation entropy changes from liquid to vapor. The near horizontal region ($P_R < 1$) represents combined two-phase and vapor flow (mist flow) and the final drop represents the transfer from two-phase to single phase flow throughout the nozzle. The horizontal line (at low pressure) represents single phase flow.

Comparison with Nitrogen and Methane Data

Choked flow data were taken for fluid nitrogen and methane in a converging-diverging axisymmetric nozzle. The complete set of nitrogen data and description of the experiment are presented in reference 6. Subsequently methane data were taken in the same facility in order to make the corresponding states comparison presented in this paper. From the data acquired for the two fluids it was possible to obtain four matching reduced isotherms for comparison. These were $T_R = 0.81, 0.88, 0.96, \text{ and } 1.035$. The range of data obtainable was dictated by the test facility which was originally a liquid hydrogen heat transfer facility. The data are presented in Table III and shown, along with the equilibrium theory curves, on figures 4 and 5.

A careful examination of figures 4 and 5 and Table III reveals that in general the nitrogen and methane experimental data reduce very well by corresponding states. If the data follow the equilibrium theory fairly well, such as along $T_R = 1.035$, then both fluids do. If the data deviates from the equilibrium theory, such as along $T_R = 0.96$, then both fluids deviate--and in the same direction and by about the same amount. As stated earlier the authors do not wish to attempt to verify any particular theory in this paper; however, some comments are in order. The data of this paper, and particularly that of reference 6, suggest that an isentropic equilibrium analysis will work well above $T_R = 1.0$. On the other hand, near saturated liquid stagnation conditions nonequilibrium effects are probably important and analyses, such as Henry and Fauske (ref. 3), will be required. The region in between is not clear yet and more experimental and analytical work is required. Substantial deviations in slope, such as the flagged data points along $T_R = 0.88$, cannot be correlated by either type theory at present, and may represent other factors such as two-dimensional effects in the nozzle.

CONCLUSIONS

Two-phase critical flow and critical flow pressure ratio of simple fluids through a nozzle have been shown to nearly obey the principle of corresponding states.

Normalizing the critical flow by

$$G^* = \sqrt{\frac{P_c P_c}{Z_c}}$$

nearly reduces the calculated equilibrium value of critical flow to a single isothermal curve for the fluids nitrogen, oxygen, methane and argon.

For dense fluids and near the thermodynamic critical point the critical flow pressure ratio, P_t/P_o , is approximately the same for each fluid as a function of reduced stagnation pressure along lines of constant reduced stagnation temperature. In the less dense or gaseous region a normalization of critical flow pressure ratio by

$$\left(\frac{P_t}{P_o} \right)_{P_c} = \left[\frac{2}{\gamma + 1} \right]^{\frac{\gamma}{\gamma - 1}}$$

seems to be required.

In the region around $T_R = 1.1$ the spread among the curves, which represents the degree of failure to follow corresponding states, is fairly large (greater than 10%). This is unexplained at present and requires further examination.

The corresponding states normalization presented is also supported in this paper by nitrogen and methane two-phase choked flow data.

Since the reduced curves for nitrogen, oxygen and methane are so close, it is felt that oxygen two-phase choked flow can be predicted using corresponding states reduction and relying on nitrogen and methane data to establish the appropriate theory to use. Consequently only a limited number of experimental oxygen points are required to verify this conclusion.

NOMENCLATURE

C	Sonic velocity	cm/sec
C _p	Specific heat at constant pressure	j/gm K
C _v	Specific heat at constant volume	j/gm K
C _{p₀} , C _{v₀}	Zero density specific heats	j/gm K
f, g	Functions	-
G	Mass flux	gm/cm ² sec
G*	Mass flux normalizing constant	gm/cm ² sec
H	Enthalpy	j/gm
P	Pressure	MN/m ²
R	Gas constant	j/gm K
T	Temperature	K
V	Specific volume	cm ³ /gm
X	Quality	-
Z	Compressibility coefficient	-
γ	Specific heat ratio C _p /C _v	-
μ	Joule-Thomson coefficient	K m ² /MN
ρ	Density	gm/cm ³

Subscripts

c	Thermodynamic critical point
g	Gas (vapor)
l	Liquid
max	Maximum
o	Stagnation
PG	Perfect gas
R	Reduced or normalized (see text)
t	Throat

FLUID	P_c MN/m ²	T_c K	ρ_c gm/cc	Z_c Z_c	G^* gm/cm ² sec
Nitrogen	3.417	126.3	.3105	.2937	6010.4
Oxygen	5.083	154.78	.4325	.2922	8673.9
Methane	4.627	190.77	.162	.2889	5093.7
Argon	4.865	150.7	.531	.2921	9404.2

Table I Critical Constants Used in the Reducing Parameters

FLUID	γ	$\left. \frac{P_t}{P_o} \right] PG$
Nitrogen	1.4	.5283
Oxygen	1.396	.5290
Methane	1.304	.5450
Argon	1.668	.4870

Table II Asymptotic Values of the Critical Pressure Ratio

TABLE III NITROGEN AND METHANE CHOKED FLOW DATA

Run	Fluid	To)R	Po)R	G _{exp})R	P _t /Po, exp)R	G _{calc})R	P _t /Po, calc)R	Flow Er- ror %	Press. Er- ror %
802	N ₂	.817	2.267	1.538	.170	1.627	.182	5.78	7.48
801		.817	1.789	1.342	.219	1.416	.244	5.49	11.51
790		.817	1.765	1.334	.221	1.405	.248	5.33	12.07
797		.817	1.068	.982	.389	1.020	.447	3.91	14.90
786		.817	1.028	.956	.406	.994	.466	3.98	14.93
983	CH ₄	.787	1.645	1.311	.207	1.371	.215	4.60	3.72
993		.808	1.594	1.242	.249	1.311	.268	5.63	7.40
992		.807	1.478	1.192	.268	1.255	.288	5.24	7.75
984		.794	1.218	1.092	.297	1.134	.322	3.79	8.73
991		.809	1.188	1.045	.346	1.090	.378	4.21	9.22
985		.801	.901	.894	.441	.918	.480	2.77	8.99
990		.813	.897	.868	.493	.892	.534	2.71	8.30
986		.807	.739	.771	.574	.782	.629	1.38	9.66
987		.813	.612	.659	.712	.653	.810	-0.89	13.79
989		.820	.591	.628	.754	.614	.889	-2.13	17.89
988		.821	.467	.521	.859	.463	1.153	-11.03	34.17

TABLE III (Continued)

Run	Fluid	To) R	Po) R	G _{exp}) R	P _t /Po, exp) R	G _{calc}) R	P _t /Po, calc) R	Flow Er- ror %	Press. Er- ror %
639	N ₂	.876	2.392	1.489	.252	1.579	.265	6.05	5.12
638		.879	1.749	1.219	.372	1.285	.401	5.43	7.79
712		.886	1.639	1.157	.422	1.217	.456	5.21	8.21
512		.876	1.611	1.165	.411	1.219	.436	4.67	6.04
711		.885	1.412	1.047	.502	1.094	.544	4.51	8.33
710		.885	1.221	.941	.587	.977	.647	3.82	10.32
637		.885	1.206	.930	.593	.967	.657	4.01	10.72
1050	CH ₄	.879	1.317	.972	.544	1.035	.566	6.55	4.09
1042		.873	1.272	.961	.549	1.035	.569	7.78	3.61
1051		.884	1.043	.806	.659	.834	.762	3.56	15.71
1043		.875	1.042	.822	.652	.860	.721	4.68	10.42
1044		.877	.895	.734	.659	.737	.868	0.39	31.65
1055		.897	.879	.725	.609	.662	1.015	-8.74	66.71
1052		.887	.800	.695	.613	.621	1.054	-10.62	71.83
1045		.880	.720	.653	.673	.563	1.127	-13.66	67.40
1053		.891	.595	.555	.770	.338	1.519	-39.13	97.38
1046		.883	.592	.556	.851	.383	1.433	-31.01	68.46
1054		.898	.540	.364	1.218	.248	1.352	-31.82	11.05
1047		.884	.491	.253	1.341	.234	1.368	-6.92	2.03

TABLE III (Continued)

Run	Fluid	To)R	Po)R	G _{exp})R	P _t /Po, exp)R	G _{calc})R	P _t /Po, calc)R	Flow Er- ror %	Press. Er- ror %
533	N ₂	.960	1.612	.997	.632	1.009	.742	1.30	17.40
534		.960	1.403	.890	.664	.874	.886	-1.74	33.39
535		.960	1.211	.794	.723	.729	1.069	-8.05	47.91
862		.960	1.103	.725	.834	.634	1.204	-12.59	44.43
536		.961	1.034	.666	.954	.555	1.323	-16.63	38.69
863		.961	.856	.484	1.324	.351	1.342	-27.52	1.32
865		.959	.746	.285	1.193	.181	1.120	-36.52	-6.09
936		.958	.743	.216	1.088	.180	1.120	-16.56	2.92
1015	CH ₄	.955	1.592	.954	.627	1.011	.710	5.99	13.20
1016		.956	1.337	.831	.668	.842	.890	1.37	33.37
1017		.955	1.062	.672	.889	.615	1.187	-8.39	33.53
1018		.955	.896	.522	1.182	.417	1.431	-20.11	21.04
1019		.955	.811	.397	1.442	.332	1.311	-16.29	-9.06

TABLE III (Continued)

Run	Fluid	To)R	Po)R	G _{exp})R	P _t /Po, exp)R	G _{calc})R	P _t /Po, calc)R	Flow Er- ror %	Press. Er- ror %
620	N ₂	1.038	2.496	1.195	.545	1.228	.611	2.71	12.01
621		1.036	2.043	.992	.706	1.008	.792	1.63	12.11
622		1.032	1.632	.770	.962	.766	1.053	-0.52	9.45
884		1.032	1.612	.764	.976	.755	1.067	-1.18	9.35
770		1.032	1.513	.701	1.077	.679	1.164	-3.28	8.03
679		1.034	1.441	.621	1.183	.606	1.253	-2.44	5.90
680		1.030	1.327	.532	1.317	.509	1.383	-4.53	4.97
682		1.031	1.317	.520	1.339	.493	1.401	-5.34	4.65
757		1.031	1.295	.500	1.375	.466	1.434	-6.79	4.30
687		1.035	1.271	.449	1.339	.414	1.476	-7.68	10.28
899	CH ₄	1.035	1.224	.404	1.188	.366	1.272	-9.32	7.06
894		1.033	1.195	.386	1.172	.347	1.239	-10.25	5.75
945		1.036	1.097	.288	1.161	.283	1.179	-1.79	1.48
1099		1.055	1.979	.862	.822	.887	.858	2.94	4.33
1100		1.045	1.713	.725	.982	.746	1.016	2.87	3.42
1101		1.039	1.470	.586	1.175	.596	1.205	1.81	2.56
1102		1.032	1.325	.488	1.327	.486	1.354	-0.40	2.09
1103		1.031	1.298	.465	1.358	.461	1.387	-0.76	2.16
1104		1.028	1.254	.430	1.236	.421	1.443	-2.10	16.73
1105		1.022	1.173	.386	1.163	.365	1.217	-5.39	4.67

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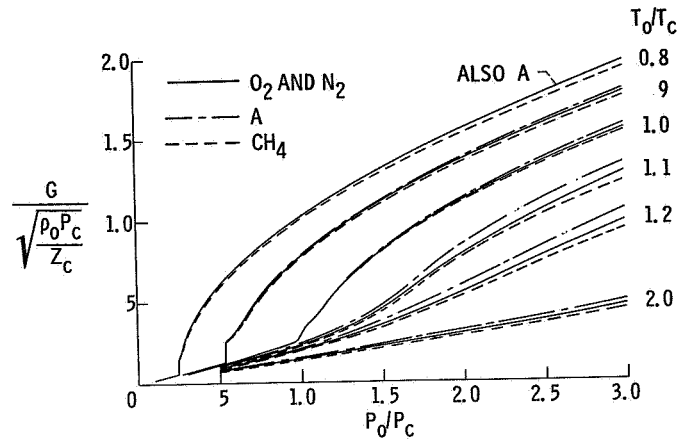


Figure 1. - Critical flow rate of oxygen, nitrogen, argon, and methane computed by isentropic equilibrium expansion using corresponding states.

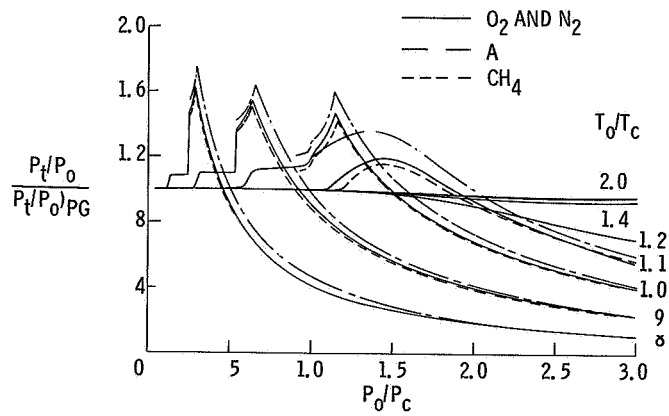


Figure 2. - Critical flow pressure ratio of oxygen, nitrogen, argon, and methane computed by isentropic equilibrium expansion using corresponding states variables, normalized by perfect gas ratio.

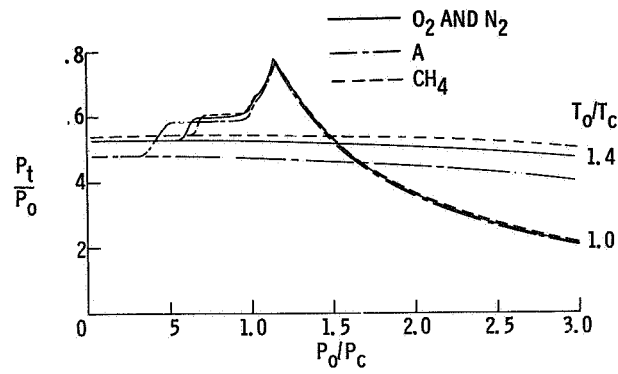


Figure 3. - Critical flow pressure ratio of oxygen, nitrogen, argon, and methane computed by isentropic equilibrium expansion using corresponding states variables.

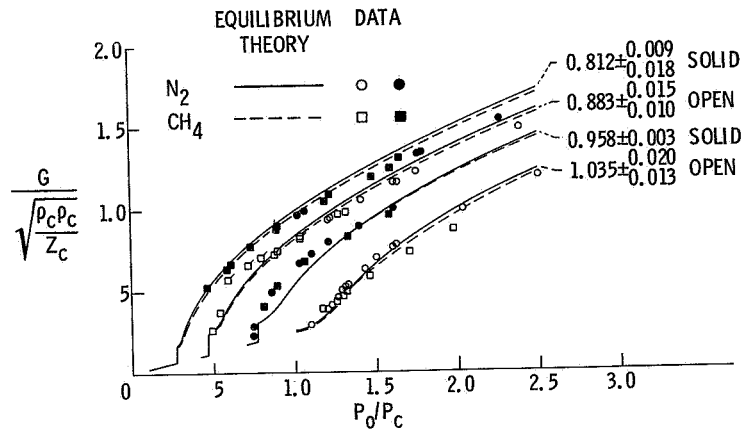


Figure 4. - Comparison of nitrogen and methane critical flow rate data on corresponding states basis.

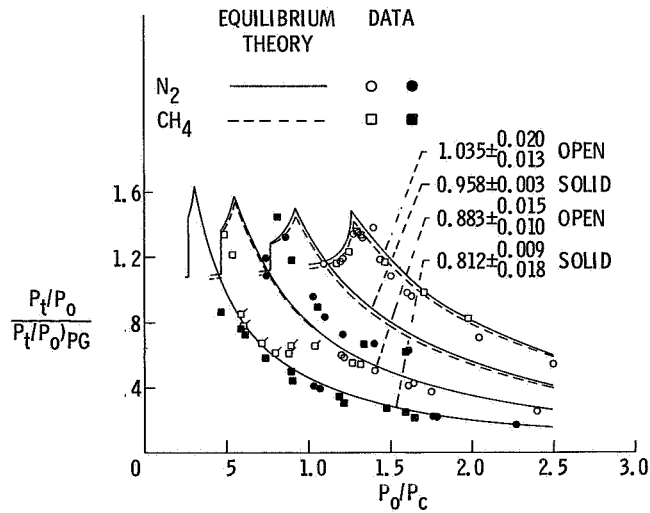


Figure 5. - Comparison of nitrogen and methane critical flow pressure ratio data on corresponding states basis.